

ROME Example:

Portfolio Allocation under the Entropic Satisficing Measure

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1 Introduction

The example is adopted from Brown and Sim [2] and Brown, De Giorgi and Sim [3]. The entropic (prospective) satisficing measure or EPSM is defined as

$$\rho(X) \triangleq \sup \left\{ k : \frac{1}{k} \ln \mathbb{E} [\exp(-Xk)] \leq 0 \right\}.$$

Note that EPSM is reciprocal to the riskiness index introduced by Aumann and Serrano [1]. In this example, we obtain the portfolio weights that optimize the EPSM objective using ROME. In particular, we illustrate the use of `expcone` function provided in ROME (see ROME manual), which is useful in modeling convex constraints involving exponential, geometric or entropic functions.

2 Model Description

Consider n assets with independent random returns V_i , $i = 1, \dots, n$ that are discretely distributed as follows:

$$\mathbb{P} \{V_i = v\} = \begin{cases} p_i & \text{if } v = \underline{v}_i \\ 1 - p_i & \text{if } v = \bar{v}_i \end{cases}$$

We then consider the following asset allocation problem

$$\begin{aligned} \sup \quad & \rho \left(\sum_{i=1}^n w_i V_i - \tau \right) \\ \text{s.t.} \quad & \sum_{i=1}^n w_i = 1 \\ & w_i \geq 0, \quad i = 1, \dots, n. \end{aligned}$$

where τ is a given target return such that $\tau < \max_i \mathbb{E}[V_i]$. We solve the following optimization problem

$$\begin{aligned}
& \max && k \\
& \text{s.t.} && \sum_{i=1}^n \frac{1}{k} \ln(p_i \exp(-w_i k \underline{v}_i) + q_i \exp(-w_i k \bar{v}_i)) \leq -\tau \\
& && \sum_{i=1}^n w_i = 1 \\
& && k > 0, w_i \geq 0, \quad i = 1, \dots, n,
\end{aligned}$$

where $q_i = 1 - p_i$. By replacing k with $1/a$, we can convert to the following convex optimization problem

$$\begin{aligned}
& \min && a \\
& \text{s.t.} && \sum_{i=1}^n a \ln(p_i \exp(-w_i \underline{v}_i/a) + q_i \exp(-w_i \bar{v}_i/a)) \leq -\tau \\
& && \sum_{i=1}^n w_i = 1 \\
& && a > 0, w_i \geq 0, \quad i = 1, \dots, n.
\end{aligned}$$

Finally, using the `expcone` function in ROME, we can transform the problem to

$$\begin{aligned}
& \min && a \\
& \text{s.t.} && a \geq p_i \quad \underbrace{a \exp((-z_i - w_i \underline{v}_i)/a)}_{\text{expcone}(-z(i) - v_{lo}(i) * w(i), a)} + q_i a \exp((-z_i - w_i \bar{v}_i)/a) \quad i = 1, \dots, n \\
& && \sum_{i=1}^n z_i \leq -\tau \\
& && \sum_{i=1}^n w_i = 1 \\
& && a > 0, w_i \geq 0, z_i \text{ free} \quad i = 1, \dots, n.
\end{aligned}$$

References

- [1] Aumann, R., and R. Serrano (2008): An Economic Index of Riskiness, *Journal of Political Economy*, 116(5), 810-836.
- [2] Brown, D. B. and M. Sim (2009): Satisficing Measures for Analysis of Risky Positions, *Management Science*, 55(1), 71-84.
- [3] Brown, D. B., De Giorgi, E. and M. Sim (2009): A Satisficing Alternative to Prospect Theory, *Working Paper*, Available at SSRN: <http://ssrn.com/abstract=1406399>.