ROME Example: The Simple Portfolio Allocation Example from "The Price of Robustness"

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1 Introduction

This example is adapted from Bertsimas and Sim [2]. It illustrates how to construct the Bertsimas and Sim's style uncertainty set and solve the robust problem using ROME.

2 Model description

We consider a portfolio construction problem consisting of n stocks. Stock i has uncertain return $\tilde{r}_i$ represented by

$$\tilde{r}_i = \mu_i + \sigma_i \tilde{z}_i, i = 1, \ldots, n$$

in which $\tilde{\boldsymbol{z}}$ is uncertain and lies in the uncertainty set proposed by Bertsimas and Sim [2],

$$\mathcal{W}_\Gamma = \{\boldsymbol{z} \ : \ \|\boldsymbol{z}\|_\infty \leq 1, \|\boldsymbol{z}\|_1 \leq \Gamma\}.$$

The parameter Γ is commonly known as the "Budget of Uncertainty".

The objective is to determine the fraction x_i of wealth to invest in stock i so as to maximize the portfolio value under ambiguity aversion.

$$\begin{array}{ll} \max & \min\limits_{\boldsymbol{z} \in \mathcal{W}_\Gamma} \sum\limits_{i=1}^{n} (\mu_i + \sigma_i z_i) x_i \\ \text{s.t.} & \sum\limits_{i=1}^{n} x_i = 1 \\ & x_i \geq 0. \end{array} \tag{1}$$

For the test case, we consider the portfolio problem with $n = 150$,

$$\mu_i = 1.15 + i\frac{0.05}{150}, \qquad \sigma_i = \frac{0.05}{450}\sqrt{2in(n+1)}.$$

Hence, stocks with higher returns are also more risky.

Note that it is also easy to modify the ROME code to consider Ben-Tal and Nemirovski's [1] uncertainty set as follows

$$\mathcal{W}_\Gamma = \{\boldsymbol{z} \ : \ \|\boldsymbol{z}\|_\infty \leq 1, \|\boldsymbol{z}\|_2 \leq \Gamma\}.$$

References

[1] Ben-Tal, A., Nemirovski, A. (2000): Robust solutions of Linear Programming problems contaminated with uncertain data, *Math. Program.*, 88, 411-424.

[2] Bertsimas, D. and M. Sim. (2004): Price of robustness. *Operations Research*, 52, 35-53.