ROME Example: Single Product Multi-Period Inventory Control

Joel Goh Melvyn Sim

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1 Introduction

This example is adapted from See and Sim [1]. Instead of using the decision rules described in the paper, we formulate the stochastic inventory control problem directly and obtain the solutions via ROME. In general, ROME's solutions are not the same as See and Sim's [1]. We believe that ROME's solutions are at most slightly worse off. Nevertheless, the model in ROME is more intuitive and leads to smaller sized problem that is faster to solve.

2 Model description

We consider a single product stochastic inventory system with T planning horizons from $t = 1$ to $t = T$. The timeline of events is as follows:

1. At the beginning of the t^{th} time period, before observing the demand, the inventory manager places an order of x_t at unit cost c_t. The product is assumed to to arrive immediately, i.e., there is no lead-time.

2. The inventory manager faces an initial inventory level y_t and receives an order of x_t. The demand for inventory within the period is realized at the end of the time period. After receiving a demand of d_t, the inventory level at the end of the period is $y_t + x_t - d_t$.

3. Excess inventory is carried to the next period incurring a per-unit overage (holding) cost. On the other hand, each unit of unsatisfied demand is backlogged (carried over) to the next period with a per-unit underage (backlogging) penalty cost. At the last period, $t = T$, the penalty of lost sales can be accounted for through the underage cost.

We introduce the following notations:

- $\tilde{d}_t$: stochastic exogenous demand at period t.
- $\tilde{\boldsymbol{d}}_t$: a vector of random demands from period 1 to t, that is, $\tilde{\boldsymbol{d}}_t = (\tilde{d}_1, \ldots, \tilde{d}_t)$.
- $x_t(\tilde{\boldsymbol{d}}_{t-1})$: order placed at the beginning of the tth time period after observing $\tilde{\boldsymbol{d}}_{t-1}$. The first period inventory order is denoted by $x_1(\tilde{\boldsymbol{d}}_0) = x_1^0$.
- $y_t(\tilde{\boldsymbol{d}}_{t-1})$: inventory level at the beginning of the t^{th} time period. The initial inventory level is denoted by $y_1(\tilde{\boldsymbol{d}}_0) = y_1^0$.
- h_t: unit inventory overage (holding) cost charged on excess inventory at the end of the t^{th} time period
- b_t: unit underage (backlog) cost charged on backlogged inventory at the end of the tth time period.
- c_t: unit purchase cost of inventory for orders placed at the t^{th} time period
- x_{max}: the maximum amount that can be ordered.

The inventory manager's objective is to determine the dynamic ordering quantities x_t from period $t = 1$ to period $t = T$ so as to minimize the total expected ordering, inventory overage (holding) and underage (backlog) costs in response to the uncertain demands. The multi-period inventory problem can be formulated as a T stage stochastic optimization model as follows:

$$\begin{aligned} \min \quad & \sum_{t=1}^{T} \left(\mathrm{E}\left(c_t x_t(\tilde{\boldsymbol{d}}_{t-1}) \right) + \mathrm{E}\left(h_t (y_{t+1}(\tilde{\boldsymbol{d}}_t))^+ \right) + \mathrm{E}\left(b_t (y_{t+1}(\tilde{\boldsymbol{d}}_t))^- \right) \right). \\ \text{s.t.} \quad & y_{t+1}(\tilde{\boldsymbol{d}}_t) = y_t(\tilde{\boldsymbol{d}}_{t-1}) + x_t(\tilde{\boldsymbol{d}}_{t-1}) - \tilde{d}_t \qquad t = 1, \ldots, T \\ & 0 \leq x_t(\tilde{\boldsymbol{d}}_{t-1}) \leq x_{max} \qquad t = 1, \ldots, T. \end{aligned} \tag{1}$$

2.1 Demand uncertainty

The demand process is given by

$$d_t(\tilde{\boldsymbol{z}}) = \tilde{z}_t + \alpha \tilde{z}_{t-1} + \alpha \tilde{z}_{t-2} + \cdots + \alpha \tilde{z}_1 + \mu, \tag{2}$$

where the factors $\tilde{z}_t$ are independent random variables with zero means. The demand process of Equation (2) for $t \geq 2$ can be expressed recursively as

$$d_t(\tilde{\boldsymbol{z}}) = d_{t-1}(\tilde{\boldsymbol{z}}) - (1 - \alpha)\tilde{z}_{t-1} + \tilde{z}_t. \tag{3}$$

As α grows, the demand process becomes non-stationary and less stable with increasing variance. When $\alpha = 1$, the demand process is a random walk on a continuous state space.

References

[1] See, CT, M. Sim. (2009): Robust Approximation to Multi-Period Inventory Management, *Operations Research forthcoming.*