

ROME Example: Warehouse Management

Joel Goh

Melvyn Sim

Sep 2009

1 Introduction

This example is adapted from Ang et al. [1]. It is a multiperiod optimization problem that can easily be modeled in ROME. Note that excessive use of loops in a Matlab program can lead to sluggish performance. This example illustrates the usefulness of Matlab functions such as `sum`, `squeeze`, `cat` and `repmat` in reducing the use of loops in the ROME code.

2 Model description

We consider a robust warehouse optimization model that minimizes the worst-case expected total operating cost of an unit-load warehouse. Let there be N classes of storage locations in the warehouse. The classes are indexed by $j = 1, \dots, N$. To simplify the analysis, we assume that the storage and retrieve costs are proportional to the number of pallets handled. Let s_j and r_j denote the unit store cost and retrieve cost of class j respectively. Assume class j has capacity c_j for $j = 1, \dots, N$. We assume that the N -th class represents emergency storage, which has infinite capacity ($c_N = \infty$) but incurs high store and retrieve costs.

Let there be M products indexed by $i = 1, \dots, M$. We divide the planning horizon into T periods, which are indexed by $t = 1, \dots, T$. For each period t , we assume that all pallets arrive at the start of the period and they are ready to be assigned to storage locations. For all pallets that are ordered during period t , we assume that they are retrieved from the warehouse upon demand request at the end of the period. The objective is to minimize the total expected operating cost of the unit-load warehouse for the entire planning horizon. To simplify notations, we let $\mathcal{N} = \{1, \dots, N\}$, $\mathcal{N}^- = \{1, \dots, N - 1\}$, $\mathcal{T} = \{1, \dots, T\}$, $\mathcal{T}^+ = \{1, \dots, T + 1\}$ and $\mathcal{M} = \{1, \dots, M\}$.

Let a_{it} denotes the number of pallets of product i that arrive at the start of period t . The number of pallets of product i at the end of the period t is given by $d_{it}(\tilde{\mathbf{z}}^t)$, where $\tilde{\mathbf{z}}^t$ contains all the uncertain factors (or information) that are revealed at the end of time period t . The distribution of $\tilde{\mathbf{z}}^T$ (or simply $\tilde{\mathbf{z}}$) is unknown but belongs to a family of distribution, \mathbb{F} .

At the start of the t th period, the pallets arrive according to a given schedule. A decision on where to store these pallets is made based on the information available captured in $\tilde{\mathbf{z}}^{t-1}$. These pallets are then moved to their assigned storage locations. After we know the demand in the period, the information available becomes $\tilde{\mathbf{z}}^t$, and a decision on where to retrieve pallets to fulfill the demand is made. Pallets are then retrieved from their storage locations.

We introduce the following adjustable or recourse variables.

1. $v_{ijt}(\tilde{\mathbf{z}}^{t-1})$: The number of arriving pallets of product i assigned to class j at the start of period t , after observing $\tilde{\mathbf{z}}^{t-1}$.
2. $w_{ijt}(\tilde{\mathbf{z}}^t)$: The number of pallets of product i retrieved from class j after observing $\tilde{\mathbf{z}}^t$. This is the decision to be made after the demand in period t is realized.
3. $x_{ij(t+1)}(\tilde{\mathbf{z}}^t)$: The number of pallets of product i in storage class j at the start of period $t + 1$.

The robust warehouse optimization model is given by

$$\begin{aligned}
Z_R = \min \quad & \max_{\mathbb{P} \in \mathbb{F}} \mathbb{E}_{\mathbb{P}} \left[\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{N}} (s_j v_{ijt}(\tilde{\mathbf{z}}^{t-1}) + r_j w_{ijt}(\tilde{\mathbf{z}}^t)) \right] \\
\text{s.t.} \quad & \sum_{j \in \mathcal{N}} v_{ijt}(\tilde{\mathbf{z}}^{t-1}) = a_{it}, & i \in \mathcal{M}, t \in \mathcal{T}; \\
& \sum_{j \in \mathcal{N}} w_{ijt}(\tilde{\mathbf{z}}^t) = d_{it}(\tilde{\mathbf{z}}^t), & i \in \mathcal{M}, t \in \mathcal{T}; \\
& x_{ij(t+1)}(\tilde{\mathbf{z}}^t) = x_{ijt}(\tilde{\mathbf{z}}^{t-1}) + v_{ijt}(\tilde{\mathbf{z}}^{t-1}) - w_{ijt}(\tilde{\mathbf{z}}^t), & i \in \mathcal{M}, j \in \mathcal{N}, t \in \mathcal{T}; \\
& x_{ij1} = 0, & i \in \mathcal{M}, j \in \mathcal{N}; \\
& \sum_{i \in \mathcal{M}} (x_{ijt}(\tilde{\mathbf{z}}^{t-1}) + v_{ijt}(\tilde{\mathbf{z}}^{t-1})) \leq c_j, & j \in \mathcal{N}^-, t \in \mathcal{T}; \\
& v_{ijt}(\tilde{\mathbf{z}}^{t-1}), w_{ijt}(\tilde{\mathbf{z}}^t) \geq 0, & i \in \mathcal{M}, j \in \mathcal{N}, t \in \mathcal{T}; \\
& x_{ijt}(\tilde{\mathbf{z}}^t) \geq 0, & i \in \mathcal{M}, j \in \mathcal{N}, t \in \mathcal{T}^+;
\end{aligned} \tag{1}$$

Class	Store cost	Retrieve cost	Capacity
1	1	1	150
2	1.5	1.5	300
3	2	2	450
4	3	3	600
5	100	100	∞

Table 1: Layout of a warehouse

2.1 An example

We consider the problem with 3 products and 5 storage classes. Table 1 shows the layout of the warehouse. Note that storage class 5 represents an emergency storage area with high store and retrieve costs. We assume the emergency storage area has an infinite capacity. The number of arriving pallets and uncertain demand for each product in each period is given by

$$d_{it}(\tilde{\mathbf{z}}^t) = \mu_{it} + \sigma_{it}\tilde{z}_{it},$$

where $\tilde{\mathbf{z}}^t$ are uncertain factors with zero means and support set

$$W = \{\mathbf{z} \mid -\mathbf{1} \leq \mathbf{z} \leq \mathbf{1}\} \cap G,$$

$$G = \left\{ \mathbf{z} \mid \sum_{\tau=1}^t d_{i\tau}(\mathbf{z}^\tau) \leq \sum_{\tau=1}^t a_{i\tau}, d_{it}(\mathbf{z}^t) \geq 0 \text{ for } i \in \mathcal{M}, t \in \mathcal{T} \right\}.$$

Note that uncertainty set G ensures that demands of products are nonnegative and do not exceed the inventory available in the warehouse. Please refer to [1] for more information on this.

2.2 Exercise

Use ROME to answer the following questions:

1. What happens if G is not part of the support set?
2. Would there be any improvement from using deflected linear decision rules?
3. Modify the program to minimize the worst case cost (as opposed to worst case *expected* cost).

References

- [1] Ang, M., YF. Lim, M. Sim. (2009): Robust Warehouse Management, *NUS Working paper*
http://www.bschool.nus.edu/STAFF/dscsim/docs/Ang_Lim_Sim-20081101.pdf