Distributionally Robust Optimization with ROME (part 2)

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Guest Lecture
Outline

Introduction

Basic Structure of a ROME Program

ROME Features

Example 1: Simple Test

Example 2: Inventory Management

Conclusion
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What is ROME?

- ROME: Robust Optimization Made Easy
- Algebraic modeling language in the MATLAB environment for modeling Robust Optimization (RO) problems
- Primarily designed for RO problems within the DRO framework
- ROAM Design Goals
  - Environment for rapid prototyping of new RO ideas
  - Ease the transition from theory to practice
  - Ease numerical studies of RO models
What ROME is NOT

- ROME is NOT a solver engine
  - ROME calls 3rd party solvers to do actual solving (e.g. CPLEX, MOSEK, SDPT3)
  - ROME serves as an intermediary to translate an uncertain optimization program from a mathematically intuitive form into a solver-understandable form

- ROME is NOT a large-scale solver platform
  - ROME can handle medium-sized problems reasonably well, and some large problems
  - Best to write specialized code (e.g. in C, C++)
Why ROME?

- RO programs, especially with more complex decision rules (e.g. BDLDR), can be extremely complex, typically involving the following steps
  - Constructing robust counterparts
  - Finding deflected components (non-anticipative)
  - Constructing robust bounds
- Fortunately, most of these steps are mechanical in nature
  - Potential to be automated
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Ingredients of a Robust Optimization Program

- Uncertainties: Methods to input distributional properties
- Variables: Deterministic variables, recourse variables
- Arithmetic operations on Uncertainties, Variables
- Conic Constraints
- Objective
- Optimization Results
The ROME Environment

- The ROME environment is started by a call to `roam_begin`
  - Sets up internal ROAM structures (invisible)
- A model in ROAM is created by a call to `roam_model`
  - Returns a handle which you can use to access model data, or call model member functions
- Input uncertainties, variables, constraints, and objective
- Call `solve` to complete the model and run solver.
- Can access optimization results after solve
- `roam_end` clears the ROME environment and frees memory in ROAM structures
Variables and Uncertainties

• Declared with the `newvar` keyword with different options
  • `<nothing>`: deterministic variables
  • `uncertain`: uncertainties
  • `linearrule>`: LDR recourse variables

• Can set distributional properties on uncertainty variables

• Can be constructed with various sizes
  • Scalars, vectors, matrices, multi-dimensional arrays
ROME Structure Code (I)

% ROME_STRUCTURE.m
% File to illustrate ROME’s structure. No real model to be solved.

% PREAMBLE
% Begins ROME Environment and creates model object
rome_begin; % Begins the ROME Environment
h = rome_model('Dummy File'); % Creates a model object

% MAIN CODE BODY
% Declare variables, objective and constraints
% Declare uncertain variables first, and set their distributional properties,
% mean, covariance, directional deviations, support. Ends with call to solve.
% Declare variables:
newvar z(5, 1) uncertain; % Declare z as 5 x 1 uncertainty vector
z.set_mean(0); % Set mean of z
z.Covar = 1; % Set covariance of z

newvar x(3, 1); % Declare 3 x 1 variable x
newvar y(3, 4, z) linearrule; % Declare 3 x 4 LDR variable y
Operators

- Most of MATLAB’s arithmetic operators have been overloaded to work with declared ROME variables
  - Addition / Subtraction
  - Array and Matrix Multiplication
  - Subscripted Reference and Assignment (e.g. A(1, 2))
  - Array shape manipulation commands (size, reshape)
- Idea: whatever you can do with MATLAB matrices, you should be able to do with ROME variables
- Non-standard operation: mean (for uncertainties and LDR variables)
Constraints and Objective

- **Constraints**
  - Declared using the `rome_constraint` function
  - Accepts ROME expressions containing a single inequality or equalities (examples later)
  - For LDR variables, constraints will be translated into the Robust Counterpart *automatically*

- **Objective**
  - One objective per model
  - Specified using `rome_minimize` or `rome_maximize`
Getting Optimization Results

- After calling `solve`,
  - Get values of variables using `eval`
  - Can get values of declared variables or even functions of variables
% Declare constraints:
  rome_constraint(x(2:3) <= 2*x(1:2));
  rome_constraint(sum(y, 1) == 1);
  rome_constraint(y(:, 1) + x >= 2);
  rome_constraint(norm2(x) <= 1);
  rome_constraint(mean(y(2, :)) <= 5);

% Declare objective:
  rome_minimize(sum(x) + sum(y(:, 2))); % objective

% Instruct ROME to solve
  h.solve; % Terminate the current model with solve

% GET RESULTS
% Get optimization results
  y_sol = h.eval(y);
  u_sol = h.eval(y(2, 2) + x(1)); % Get y and function of variables

% CLEANUP
% Clear up ROME memory
  rome_end; % Complete modeling and deallocate ROME memory
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Why Use ROME?

- Uncertainty description
- LDRs as primitives variables in ROAM
- Non-anticipative requirements
- In-built support for BDLDRs
- Numerical analysis of results
Uncertainty Description

- Can set distributional properties of uncertainties
- Stored in ROAM’s memory and invoked when necessary
  - e.g. Robust Counterpart for inequalities on LDRs
  - e.g. Robust Bounds

```matlab
newvar z(10, 1) uncertain; % create 10 x 1 uncertainty vector

rome_constraint(z.mean == 1); % Sets the mean to be exactly 1

rome_constraint(z >= 0); % Sets the support of z to be
rome_constraint(z <= 2); % between 0 and 2 component-wise

z.Covar = eye(10); % Sets an identity covariance matrix

z.FDev = 1.8*ones(10, 1); % Sets the forward deviation
z.BDev = 1.8*ones(10, 1); % Sets the backward deviation
```
LDRs as Primitive Objects

- LDRs are declared and manipulated directly
  - Much more mathematically meaningful
  - Don’t have to worry about internal structure of LDR or worry about how to write the robust counterpart
  - Works in concert with the uncertainty description

```matlab
newvar z(5) uncertain ;  % make uncertainty vector
newvar y1(z) linearrule;  % create scalar LDR
newvar y2(3, z) linearrule;  % create 3 x 1 LDR
newvar y3(4, 3, z) linearrule;  % create 4 x 3 LDR
```
Non-anticipative Requirements

- In ROME, you can make variables with a prescribed dependency pattern on the uncertainties

```matlab
newvar z(N) uncertain;
y = [];
for ii = 1:N
    newvar tmp(1, z(1:ii)) linearrule;
y = [y; tmp];
end
```

- e.g. $\tilde{z}$ where the $i^{th}$ component depends on the first $i$ components of uncertainty

- More directly, can specify dependency pattern at creation

```matlab
pY = [true(N, 1), tril(true(N, N))];
newvar z(N) uncertain;
newvar y(N, z, 'Pattern', pY) linearrule;
```
In-built-support for BDLDRs

- As we saw before, BDLDRs improved over LDRs, but were terribly complicated, especially for the non-anticipative case. Recall the steps involved:
  1. Form and solve the two sub-problems
  2. Remove “unnecessary” inequality constraints
  3. Make the BDLDR and construct robust bounds
  4. Solve the final problem

- Design Concept: while the BDLDR is complex, it’s just another decision rule, you shouldn’t have to change your model to use the BDLDR

- Just call solve_deflected, instead of solve
Numerical Analysis of Results (I)

- For deterministic optimization software packages this is trivial
- Recall that recourse decisions are *functions* of uncertainties
- Most general setting in ROME: deflected variable

\[ \hat{x}(\tilde{z}) = (x^0 + X\tilde{z}) + P (y^0 + Y\tilde{z})^- \]

- eval function returns an object, which encodes these parameters.
- Use linearpart and deflectedpart functions
Numerical Analysis of Results (II)

- ROME also includes a “prettyprint” functionality to aid debugging and prototyping of small problems, e.g.

\[
y_{\text{sol}} = -0.000 + 1.000 \times z_1 + 1.00(-0.000 + 1.000 \times z_1)\, - \, 1.00(1.000 - 1.000 \times z_1)\, -
\]

- Instantiate solution of uncertainty values using `insert`
- Use as a prescriptive tool or for Monte-Carlo simulation

```matlab
x_{\text{sol}} = h.eval(x); \quad \% \text{Get the solution object}
x_{\text{vals}} = \text{zeros}(M, 100); \quad \% \text{Allocate output matrix}
\text{for } ii = 1:100
    \% \text{Instantiate solution with r.v.}
x_{\text{vals}}(:, ii) = x_{\text{sol}}.insert(z_{\text{vals}}(:, ii));
\end{\text{for}}
```
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Recall Motivating Problem for BDLDR

\[
\begin{align*}
\min_{y(\cdot)} & \quad \sup_{P \in F} \mathbb{E}_P (|y(\tilde{z}) - \tilde{z}|) \\
0 & \leq y(\tilde{z}) \leq 1 \\
y & \in \mathcal{Y}(1,1,\{1\}) \\
\quad \uparrow
\min_{y(\cdot),u(\cdot),v(\cdot)} & \quad \sup_{P \in F} \mathbb{E}_P (u(\tilde{z}) + v(\tilde{z})) \\
s.t. & \quad u(\tilde{z}) - v(\tilde{z}) = y(\tilde{z}) - \tilde{z} \\
0 & \leq y(\tilde{z}) \leq 1 \\
u(\tilde{z}), v(\tilde{z}) & \geq 0 \\
y, u, v & \in \mathcal{Y}(1,1,\{1\})
\end{align*}
\]

- Where we have used the identities \( x = x^+ - x^- , |x| = x^+ + x^- \).
ROME Code for Simple Example (I)

```matlab
% SIMPLE_EXAMPLE.m
% Script to demonstrate several key features and functions in ROME,
% and illustrate overall structure of a ROME program

% ROME MODELING CODE
% Preamble
rome_begin;
% Begins the ROME Environment
h = rome_model('Simple Example');
% Creates a model object

% Feature 1: Handling Uncertainties
newvar z uncertain;
% Declare z as a scalar uncertainty
z.set_mean(0);
% Set distributional properties of z
z.Covar = 1;
% zero mean and unit variance.

% Feature 2: Declaring LDRs as primitive objects
newvar u(z) v(z) linearrule nonneg; % nonnegative LDRs u and v
newvar y(z) linearrule; % LDR y

min y(·), u(·), v(·)
s.t. sup_{P \in F} (u(\tilde{z}) + v(\tilde{z}))
    u(\tilde{z}) - v(\tilde{z}) = y(\tilde{z}) - \tilde{z}
    0 \leq y(\tilde{z}) \leq 1
    u(\tilde{z}), v(\tilde{z}) \geq 0
    y, u, v \in \mathcal{Y}(1, 1, \{1\})
```

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DRO with ROME (part 2)  
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ROME Code for Simple Example (II)

```matlab
% Objective
rome_minimize(mean(u + v));

% Constraints
rome_constraint(u - v == y - z); % equality constraint
rome_box(y, 0, 1); % 0 <= y <= 1 constraint

% Feature 4: Support for BDLDRs
h.solve_deflected;
% h.solve; % Use this instead to use basic LDRs as decision rules

% Feature 5: Extract solution for analysis
y_sol = h.eval(y);
% Call ‘eval’ after solve and before rome_end

% Complete modeling and deallocate ROME memory
rome_end;
```

\[
\min_{y(\cdot), u(\cdot), v(\cdot)} \sup_{P \in \mathcal{P}} (u(\tilde{z}) + v(\tilde{z})) \\
\text{s.t.} \quad u(\tilde{z}) - v(\tilde{z}) = y(\tilde{z}) - \tilde{z} \\
0 \leq y(\tilde{z}) \leq 1 \\
u(\tilde{z}), v(\tilde{z}) \geq 0 \\
y, u, v \in \mathcal{Y}(1, 1, \{1\})
\]
Solution

$$y_{sol} = -0.000 + 1.000\times z1 + 1.00(-0.000 + 1.000\times z1)^- - 1.00(1.000 - 1.000\times z1)^-$$

$$\hat{y}_{sol}(\tilde{z}) = \tilde{z} + \tilde{z}^- - (1 - \tilde{z})^+$$

$$= \tilde{z}^+ - (1 - \tilde{z})^+$$

$$= \begin{cases} 
\tilde{z} & \text{if } 0 \leq \tilde{z} \leq 1 \\
0 & \text{if } \tilde{z} \leq 0 \\
1 & \text{if } \tilde{z} \geq 1 
\end{cases}$$
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### Robust Inventory Model

- Model a distribution-free, multi-period, inventory control problem with service constraints.
- Demand is exogenous, with unknown, but partially characterized distribution in a family $\mathcal{F}$, defined by:
  - Covariance Matrix: Temporal demand correlation can be modeled by a non-diagonal covariance matrix.
  - Mean: Assume fixed mean (for simplicity).
  - Support: Maximum demand in each period.
- Backorders allowed, but in some applications, a penalty cost might not be a good model for stockouts.
  - In our model, we avoid stockouts with a constraint on the fill-rate.
  - Fill-rate constraint acts as a service guarantee to the consumers.
Model Parameters

**Parameters**
- Num Periods: \( T \in \mathbb{N} \)
- Order Cost: \( c \in \mathbb{R}^T \)
- Holding Cost: \( h \in \mathbb{R}^T \)
- Min Fill Rate: \( \beta \in \mathbb{R}^T \)
- Max Order Qty: \( x^{MAX} \in \mathbb{R}^T \)

**Uncertainties**
- Demand: \( \tilde{z} \in \mathbb{R}^T \)

**Decisions**
- Order Quantity: \( x(\tilde{z}): x_t(\tilde{z}) \in \mathcal{L}(1, T, [t - 1]) \)
- Inventory Level: \( y(\tilde{z}): y_t(\tilde{z}) \in \mathcal{L}(1, T, [t]) \)
Robust Fill Rate Constraint

\[ \text{Fill Rate} = \frac{\text{Expected Sales}}{\text{Expected Demand}} \geq \beta \]

- Using our notation, the robust (worst-case) version:

\[ \inf_{P \in \mathcal{F}} E_P \left( \min \{ \tilde{z}_t, y_{t-1}(\tilde{z}) + x_t(\tilde{z}) \} \right) \geq \beta_t \mu_t \]

- Apply inventory balance equation and re-arrange:

\[ \sup_{P \in \mathcal{F}} E_P \left( y_t(\tilde{z})^- \right) \leq (1 - \beta_t) \mu_t \]
Model Formulation

- **Family of uncertainties:**

\[ F = \left\{ P : E_P (\tilde{z}) = \mu, E_P (\tilde{z} \tilde{z}^\prime) = \Sigma + \mu \mu^\prime, P \left( 0 \leq \tilde{z} \leq z^{MAX} \right) = 1 \right\} \]

- **Robust Inventory Model with Fill Rate constraints:**

\[
\begin{align*}
\min_{x(\cdot), y(\cdot)} & \quad \sup_{P \in F} \left( c' x(\tilde{z}) + h' (y(\tilde{z}))^+ \right) \\
\text{s.t.} & \quad \sup_{P \in F} \left( y(\tilde{z})^- \right) \leq (I - \text{diag}(\beta)) \mu \\
& \quad D y(\tilde{z}) - x(\tilde{z}) = -\tilde{z} \\
& \quad 0 \leq x(\tilde{z}) \leq x^{MAX} \\
& \quad x_t \in \mathcal{L}(1, T, [t - 1]) \quad \forall t \in [T] \\
& \quad y_t \in \mathcal{L}(1, T, [t]) \quad \forall t \in [T] \\
& \quad D = \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ -1 & 1 & 0 & \ldots & 0 \\ 0 & -1 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \end{bmatrix}
\end{align*}
\]
Model Transformed into DRO Framework

- After linearizing piecewise-linear terms,

\[
\begin{align*}
\min_{x(\cdot), y(\cdot), r(\cdot), s(\cdot)} \quad & \sup_{P \in \mathcal{F}} \left( c' x(\tilde{z}) + h' r(\tilde{z}) \right) \\
\text{s.t.} \quad & \sup_{P \in \mathcal{F}} (s(\tilde{z})) \leq (I - \text{diag} (\beta)) \mu \\
& r(\tilde{z}) \geq y(\tilde{z}) \\
& s(\tilde{z}) \geq -y(\tilde{z}) \\
& r(\tilde{z}), s(\tilde{z}) \geq 0 \\
& Dy(\tilde{z}) - x(\tilde{z}) = -\tilde{z} \\
& 0 \leq x(\tilde{z}) \leq x^{MAX} \\
& x_t \in \mathcal{L}(1, T, [t - 1]) \quad \forall t \in [T] \\
& r_t, s_t, y_t \in \mathcal{L}(1, T, [t]) \quad \forall t \in [T]
\end{align*}
\]
ROME Code for Inventory Problem

% model parameters
T = 10;               % planning horizon
q = 1 * ones(T, 1);  % order cost rate
hcost = 2 * ones(T, 1);  % holding cost rate
beta = 0.50 * ones(T, 1);  % minimum fillrate in each period
xMax = 100 * ones(T, 1);  % maximum order quantity in each period
alpha = 0.5;          % temporal autocorrelation factor
L = alpha * triu(ones(T), -1) + eye(T);  % autocorrelation matrix

% numerical uncertainty parameters
zMax = 105 * ones(T, 1);  % maximum demand in each period
zMean = 30 * ones(T, 1);  % mean demand in each period
zCovar = 20 * (L * L');  % temporal demand covariance

% differencing matrix
D = eye(T) - diag(ones(T-1, 1), -1);

% dependency structure
pX = logical([tril(ones(T)), zeros(T, 1)]);
ROME Code for Inventory Problem (cont.)

% Step 3: BDLDR Method
% ----------------------
h = rome_begin('Robust Inventory (BDLDR)'); tic;

% declare uncertainties
newvar z(T) uncertain nonneg;

% define uncertainty parameters
rome_constraint(z <= zMax); % support
z.set_mean(zMean); % mean
z.Covar = zCovar; % covariance

% define LDR variables
newvar x(T, z, 'Pattern', pX) linearrule; % order quantity
newvar y(T, z) linearrule; % inventory level

% define auxiliary variables
newvar r(T, z) s(T, z) linearrule nonneg;

\[
\begin{align*}
F &= \left\{ \begin{array}{l}
\mathbb{P}(0 \leq \tilde{z} \leq z^{MAX}) = 1 \\
\mathbb{P}(\tilde{z}) = \mu, \\
\mathbb{E}(\tilde{z}) = \Sigma + \mu\mu'
\end{array} \right. \\
r(\tilde{z}), s(\tilde{z}) \geq 0 \\
x_t \in \mathcal{L}(1, T, [t - 1]) \quad \forall t \in [T] \\
r_t, s_t, y_t \in \mathcal{L}(1, T, [t]) \quad \forall t \in [T]
\end{align*}
\]
ROME Code for Inventory Problem (cont.)

```matlab
%% auxiliary constraints
rome_constraint(r >= y); % since r >= y^+
rome_constraint(s >= -y); % since s >= y^-

%% fillrate constraint
rome_constraint(mean(s) <= diag(ones(T, 1) - beta) * zMean);

%% inventory balance constraint
rome_constraint(D*y == x - z);

%% order quantity constraints
rome_box(x, 0, xMax);

%% objective
rome_minimize(c'*mean(x) + hcost'*mean(r));

%% solve and display optimal objective
h.solve_deflected;
disp(sprintf('BDLDR Obj = %0.2f, time = %0.2f secs', h.ObjVal, toc));
x_sol_bdl = h.eval(x)

rome_end;
```

\[
\begin{align*}
\min_{x(\cdot), y(\cdot), r(\cdot), s(\cdot)} & \quad c'(x(\tilde{z}) + h'r(\tilde{z})) \\
\text{s.t.} & \quad \sup_{P \in \mathcal{F}} E_P (s(\tilde{z})) \leq (I - \text{diag}(\beta))\mu \\
& \quad Dy(\tilde{z}) - x(\tilde{z}) = -\tilde{z} \\
& \quad r(\tilde{z}) \geq y(\tilde{z}) \\
& \quad s(\tilde{z}) \geq -y(\tilde{z}) \\
& \quad 0 \leq x(\tilde{z}) \leq x_{\text{MAX}}
\end{align*}
\]
Numerical Example (I)

- Use a covariance matrix which represents temporal autocorrelation of demand, $\Sigma = \sigma L(\alpha)L(\alpha)'$

$$L(\alpha) = \begin{bmatrix}
1 & 0 & 0 & \ldots & 0 \\
\alpha & 1 & 0 & \ldots & 0 \\
\alpha & \alpha & 1 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\alpha & \alpha & \alpha & \ldots & 1
\end{bmatrix}$$

- $(z^{MAX}, \beta) = (100, 0.5)$

<table>
<thead>
<tr>
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<th>$\mu$</th>
<th>$\alpha$</th>
<th>$Z_{LDR}^{(2)}$</th>
<th>$Z_{LDR}$</th>
<th>$Z_{BDLDR}$</th>
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Numerical Example (II)

- \((z^{MAX}, \beta) = (200, 0.5)\)

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<th>(\alpha)</th>
<th>(Z_{LDR}^{(2)})</th>
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<th>(h)</th>
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- \((z^{MAX}, \beta) = (200, 0.8)\)

<table>
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<th>(Z_{LDR})</th>
<th>(Z_{BDLDR})</th>
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<td>163.4</td>
<td>129.5</td>
<td>3</td>
<td>10</td>
<td>0.00</td>
<td>+\infty</td>
<td>276.6</td>
<td>185.8</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>0.20</td>
<td>+\infty</td>
<td>285.3</td>
<td>136.0</td>
<td>3</td>
<td>10</td>
<td>0.20</td>
<td>+\infty</td>
<td>588.2</td>
<td>202.4</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>0.50</td>
<td>+\infty</td>
<td>722.1</td>
<td>161.5</td>
<td>3</td>
<td>10</td>
<td>0.50</td>
<td>+\infty</td>
<td>1681.6</td>
<td>267.8</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>0.90</td>
<td>+\infty</td>
<td>3402.0</td>
<td>2563.5</td>
<td>3</td>
<td>10</td>
<td>0.90</td>
<td>+\infty</td>
<td>9192.7</td>
<td>7019.9</td>
</tr>
</tbody>
</table>
ROME Output (MOSEK Solver)

EDU>> inventory_fillrate_example
Status: OPTIMAL
LDR Obj = 1747.50, time = 0.25 secs

x_sol_ldr =

97.500
0.000 + 0.952*z1
-0.000 + 0.007*z1 + 0.945*z2
 0.000 + 0.005*z1 + 0.010*z2 + 0.937*z3
-0.000 + 0.004*z1 + 0.006*z2 + 0.013*z3 + 0.929*z4
 0.000 + 0.003*z1 + 0.005*z2 + 0.008*z3 + 0.015*z4 + 0.921*z5
-0.000 + 0.004*z1 + 0.004*z2 + 0.006*z3 + 0.009*z4 + 0.018*z5 + 0.911*z6
-0.000 + 0.003*z1 + 0.005*z2 + 0.006*z3 + 0.008*z4 + 0.012*z5 + 0.023*z6 + 0.897*z7
 0.000 + 0.004*z1 + 0.005*z2 + 0.006*z3 + 0.009*z4 + 0.011*z5 + 0.016*z6 + 0.031*z7 + 0.870*z8
-0.000 + 0.007*z1 + 0.007*z2 + 0.009*z3 + 0.011*z4 + 0.015*z5 + 0.019*z6 + 0.031*z7 + 0.056*z8 + 0.795*z9

Status: NEAR_OPTIMAL
BDLDR Obj = 300.48, time = 0.58 secs

x_sol_bdlr =

15.333
-0.000 + 1.003*z1 + 1.00(-0.000 + 1.003*z2)^- - 1.00(100.000 - 1.003*z1)^- -
-0.000 + 0.000*z1 + 1.004*z2 + 1.00(-0.000 + 0.000*z1 + 1.004*z2)^- - 1.00(100.000 - 0.000*z1 - 1.004*z2)^- -
-0.000 + 0.000*z1 - 0.000*z2 + 1.004*z3 + 1.00(-0.000 + 0.000*z1 - 0.000*z2 + 1.004*z3)^- - 1.00(100.000 - 0.000*z1 - 1.004*z2)^- -
-0.000 - 0.000*z1 - 0.000*z2 + 0.000*z3 + 1.004*z4 + 1.00(-0.000 - 0.000*z1 - 0.000*z2 + 0.000*z3 + 1.004*z4)^- -
 0.000 - 0.000*z1 - 0.000*z2 - 0.000*z3 + 0.000*z4 + 1.004*z5 + 1.00(0.000 - 0.000*z1 - 0.000*z2 - 0.000*z3 + 1.004*z5)^- -
-0.000 + 0.000*z1 - 0.000*z2 + 0.000*z3 + 0.000*z4 - 0.000*z5 + 1.004*z6 + 1.00(-0.000 + 0.000*z1 - 0.000*z2 + 0.000*z3 + 0.000*z4 - 0.000*z5 + 1.004*z6)^- -
 0.000 - 0.000*z1 - 0.000*z2 - 0.000*z3 - 0.000*z4 - 0.000*z5 - 0.000*z6 + 1.004*z7 + 1.00(0.000 - 0.000*z1 - 0.000*z2 - 0.000*z3 - 0.000*z4 - 0.000*z5 - 0.000*z6 + 1.004*z7)^- -
-0.000 + 0.000*z1 - 0.000*z2 + 0.000*z3 + 0.000*z4 + 0.000*z5 - 0.000*z6 - 0.000*z7 + 1.004*z8 + 1.00(-0.000 + 0.000*z1 - 0.000*z2 + 0.000*z3 + 0.000*z4 + 0.000*z5 - 0.000*z6 - 0.000*z7 + 1.004*z8)^- -
-0.000 + 0.000*z1 + 0.000*z2 - 0.000*z3 + 0.000*z4 + 0.000*z5 + 0.000*z6 + 0.000*z7 + 0.000*z8 + 1.004*z9 + 1.
Outline

Introduction

Basic Structure of a ROME Program

ROME Features

Example 1: Simple Test

Example 2: Inventory Management

Conclusion
Summary

- We have seen how ROME can be used to model Distributionally Robust Optimization problems
- ROME contains some components common to most modeling languages, and unique features for Robust Optimization
- Service-constrained inventory control example